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ABSTRACT

Data from free turbulent jets both with and without swirl are used to assess the performance of the pressure-strain model of Speziale, Sarkar and Gatski which is quadratic in the Reynolds stresses. Comparative predictions are also obtained with the two versions of the Launder, Reece and Rodi model which are linear in the same terms. All models are used as part of a complete second-order closure based on the solution of differential transport equations for each non-zero component of $\overline{u_i u_j}$ together with an equation for the scalar energy dissipation rate. For non-swirling jets, the quadratic model underestimates the measured spreading rate of the plane jet but yields a better prediction for the axisymmetric case without resolving the plane jet/round jet anomaly. For the swirling axisymmetric jet, the same model accurately reproduces the effects of swirl on both the mean flow and the turbulence structure in sharp contrast with the linear models which yield results that are in serious error. The reasons for these differences are discussed.

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1 INTRODUCTION

Swirling flows are common in nature as well as in engineering practice (e.g. in tornados, gas-turbine combustors, furnaces and jet mixers) where a tangential (swirl) velocity is often superimposed on an axially-directed flow to enhance the rate at which it spreads into and entrains from its surroundings. The generic swirling flow (i.e., the simplest realization of that class which embodies all the essential physics but none of the unnecessary geometric complexities) is that of a single axisymmetric swirling jet discharged into infinite, stagnant surroundings. For small values of the Swirl Number S (defined as the ratio of tangential to axial momentum fluxes), the adverse pressure gradients set up by the decaying tangential velocity are insufficient to cause flow reversal and the swirling jet becomes an example of a thin shear layer distorted by the imposition of an extra rate of strain. Flows of this type provide ideal benchmark tests for turbulence closures since their behavior is determined more by turbulent transport than by pressure effects and they can be simulated by the inherently more accurate marching-integration methods.

The prediction of the free swirling jet has highlighted defects in nearly all current closure models. The sensitivity of the turbulence in a thin shear layer to streamline curvature is not captured by eddy-viscosity models which utilize the Boussinesq linear stress-strain relationship. This is especially true for swirling jets where the streamlines follow helical paths. Ad-hoc corrections to various two-equation models have been reported in the literature (e.g. Rodi 1979 who modified the k-ε model), but these modifications rarely perform well in flows other than those which were used in their calibration. Adoption of more refined non-linear relationships such as that of Speziale (1987) are unlikely to lead to drastically improved predictions since they too involve a scalar eddy-viscosity while experiments show this quantity to be highly anisotropic. Second-order closure models, however, are known to reproduce the effects of two-dimensional (longitudinal) streamline curvature quite accurately (Irwin and Arnot-Smith 1975, Gibson et al. 1981) and are therefore potentially more suited for swirling flows. The first attempt at the prediction of the free swirling jet with a model of this kind appears to be that of Launder and Morse (1979). Those workers employed the more complete of the two models for the pressure-strain correlation proposed by Launder, Reece and Rodi (1975) (hereafter LRR1) and found that their predictions of the swirling jet experiments of Morse (1980) produced predominantly negative values for the shear-stress component $\overline{uw}$ while the data showed this quantity to be largely positive. This component contributes to the rate of production of $\overline{uv}$ (the shear stress component which governs the radial transport of axial momentum) which, as a result, was also underestimated by the predictions. Not surprising, therefore, Launder and Morse reported that compared to a jet without swirl, the "numerical solutions display a reduced rate of spread in contrast to the strong augmentation found in practice." Launder and Morse identified the source of this erroneous result as being due to the mean-strain contribution to the pressure-strain correlation model and demonstrated that improved predictions can be obtained by reducing these terms in the $\overline{uw}$ and $\overline{vw}$ equations by 40%. Younis (1984) found that the alternative pressure-strain model of Launder et al. (1975) (sometimes called the IP model but hereafter referred to as LRR2) suffered the same defects as its more complicated counterpart but that the model can be sensitized to swirl by treating the non-gradient terms that arise from the transformation of the convective terms into cylindrical coordinates as 'production' and including these in the pressure-strain model. This treatment renders the model dependent on the choice of the coordinate system and cannot therefore be recommended for general applications. Gibson and

Younis (1986) obtained satisfactory predictions with the coordinate-invariant form of the LRR2 model by reducing the relative importance of the mean-strain contribution. This was achieved by halving the value of the coefficient C_2, and then using well-established criteria to re-optimize the remaining coefficients. Thus, for example, the value of C_1, the coefficient multiplying the Rotta return-to-isotropy term, was increased from 1.8 to 3.0 in order to keep the value of the important parameter $(1 - C_2)/C_1$ within acceptable limits. The resulting model gave the correct results for a wide variety of complex shear flows including swirling jets, while its performance in benchmark homogeneous and inhomogeneous turbulent flows remained comparable to that of the original model. However, one adverse consequence of moving C_1 farther away from unity is the increased reliance on the model for wall-reflection effects to obtain the correct relative Reynolds-stress levels in the equilibrium near-wall layer (see Abid and Speziale 1993). C_1' and C_2' (the coefficients associated with the wall-reflection model) were therefore increased to 0.75 and 0.5 from their original values of 0.5 and 0.3, respectively. This practice of giving more prominence to the wall-reflection model seems to run counter to recent developments in the field which have led to the abandonment of this model altogether in the calculation of wall-bounded flows. Finally, in a variation on Younis' (1984) coordinate-dependent treatment discussed above, Fu, Launder and Leschziner (1987) proposed pressure-scrambling the entire convection terms and not just their non-gradient elements thus giving rise to the implausible implication that the fluctuating pressure field is modified, to first order, by the transport of $\overline{u_i u_j}$ by the mean flow. Recently, this model has been the subject of detailed analysis (Younis, Speziale and Gatski 1994) where it was shown to amount to no more than the original LRR2 formulation but with the mean-strain part omitted altogether and Rotta's coefficient C_1 increased to an unreasonably high value. The need to include the mean-strain part in the model for the pressure-strain correlation is well documented, especially when the effects of buoyancy and streamline curvature are to be properly modeled. It is therefore unlikely that this model will prove adequate when such effects are present.

Since both pressure-strain models of Launder et al. (1975) are linear in the Reynolds stresses, it seems worthwhile to investigate whether non-linear models would lead to improved predictions of the swirling jet. Several such models have been proposed in the literature. Recent models include that of Speziale, Sarkar and Gatski (1991) (hereafter SSG), which is quadratic in the Reynolds stresses, and that of Fu, Launder and Tselepidakis (1987) which is cubic in the same terms. There are no reported results with the latter model in any wall-bounded flow and since our interest is in advancing a model which would ultimately be applicable to the confined swirling flows encountered in practical applications, it seemed best to confine our attention to the quadratic model which has already been been found to reproduce the correct behavior of a number of complex wall flows without the use of empirical wall-reflection terms (Younis, Gatski and Speziale, 1994). The main objective of this work then is to see whether the presence of the quadratic terms in the SSG model provides the "major changes needed in modeling the mean-strain contribution to the pressure strain correlation" deemed necessary by Launder and Morse (1979) to resolve the swirling jet problem. The paper also reports comparative predictions with both the LRR1 and LRR2 models and puts on record the performance of the SSG model in two benchmark flows – namely, the two-dimensional plane and axisymmetric jets without swirl.

2 THE TURBULENCE MODELS

The turbulence models employed in this study are based on the solution of differential transport equations for the Reynolds-stress tensor $\overline{u_i u_j}$ of the form:

$$
\begin{aligned}
U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = & - \left(\overline{u_i u_k} \frac{\partial U_j}{\partial x_k} + \overline{u_j u_k} \frac{\partial U_i}{\partial x_k} \right) \\
& - \frac{\partial}{\partial x_k} \left[\overline{u_i u_j u_k} + \frac{1}{\rho} (\overline{p u_i} \delta_{jk} + \overline{p u_j} \delta_{ik}) - \nu \frac{\partial \overline{u_i u_j}}{\partial x_k} \right] \\
& - 2\nu \left(\overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} \right) + \overline{\frac{p}{\rho} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)}
\end{aligned} \tag{1}
$$

In the above equation, U_k is the mean velocity, u_k is the fluctuating velocity, p is the fluctuating pressure, ρ is the density and ν is the kinematic viscosity. An overbar denotes a time average.

Diffusion is assumed to be entirely due to the turbulent velocity fluctuations which are modeled here by the Daly and Harlow (1970) gradient-transport hypothesis:

$$
- \overline{u_i u_j u_k} = \frac{\partial}{\partial x_k} \left(C_s \frac{k}{\varepsilon} \overline{u_k u_l} \frac{\partial \overline{u_i u_j}}{\partial x_l} \right) \tag{2}
$$

In (2), k and ε are the turbulent kinetic energy and its dissipation rate, respectively. The coefficient C_s is assigned the value of 0.22, arrived at by Morse (1980) by computer optimization carried out in conjunction with the LRR1 model. The sensitivity of the SSG model to the choice of this value will be checked in the next section. Demuren and Sarkar (1993) obtained better Reynolds-stress anisotropies at the center-line of a channel by using the diffusion model of Mellor and Herring (1973), but experiences with the same model in free shear flows have not shown it to be superior to the simpler Daly and Harlow proposal.

The pressure-strain models employed in this paper are:

LRR1 & LRR2 Models

$$
\Phi_{ij} = -C_1 \varepsilon b_{ij} + C_2 k S_{ij} + C_3 k \left(b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} b_{kl} S_{kl} \delta_{ij} \right) + C_4 k (b_{ik} W_{jk} + b_{jk} W_{ik}) \tag{3}
$$

where

$$
LRR1 : C_1 = 3.0,\ C_2 = \frac{4}{5},\ C_3 = 1.75,\ C_4 = 1.31
$$

$$
LRR2 : C_1 = 3.6,\ C_2 = \frac{4}{5},\ C_3 = 1.20,\ C_4 = 1.20
$$

and b_{ij}, S_{ij} and W_{ij} are, respectively, the Reynolds-stress anisotropy, mean rate of strain and mean vorticity tensors defined as:

$$
b_{ij} = \frac{\overline{u_i u_j}}{\overline{u_q u_q}} - \frac{1}{3} \delta_{ij}
$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$$

$$W_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$$

SSG Model

$$\begin{aligned}\Phi_{ij} \;=\; & -(C_1\varepsilon + C_1^*\mathcal{P})b_{ij} + C_2\varepsilon\left(b_{ik}b_{kj} - \frac{1}{3}b_{kl}b_{kl}\delta_{ij}\right) + (C_3 - C_3^* II_b^{\frac{1}{2}})kS_{ij} \\ & + C_4 k\left(b_{ik}S_{jk} + b_{jk}S_{ik} - \frac{2}{3}b_{kl}S_{kl}\delta_{ij}\right) + C_5 k(b_{ik}W_{jk} + b_{jk}W_{ik})\end{aligned} \quad (4)$$

where

$$C_1 = 3.4,\ C_1^* = 1.80,\ C_2 = 4.2$$

$$C_3 = \frac{4}{5},\ C_3^* = 1.30,\ C_4 = 1.25$$

$$C_5 = 0.40,\ II_b = b_{ij}b_{ij},\ \mathcal{P} = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$$

The SSG model was formulated using a dynamical systems approach; it is topologically the generic form of the commonly used hierarchy of pressure-strain models for two dimensional mean turbulent flows that are homogeneous and in equilibrium. There are three main features that distinguish it from the LRR models: (a) the presence of a quadratic slow term (with coefficient C_2), (b) a production-based rapid term that supplements the linear part of the slow pressure strain (with coefficient C_1^*) and (c) a variable isotropic rapid term (with coefficient C_3^*). It should also be mentioned that the SSG model was optimized for the description of homogeneous turbulent flows that have combinations of rotational and irrotational strains with the result that it outperforms the LRR models in rotating homogeneous shear flow (Speziale, Sarkar and Gatski 1991). There is, of course, a strong analogy between flows with a system rotation and those with swirl and this suggests that the effects of the latter may well be better represented by the SSG model.

Closure of the $\overline{u_i u_j}$ equation is completed with the assumption that the dissipation is isotropic at high turbulence Reynolds numbers with the turbulent dissipation rate ε (where $\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}$) obtained from the solution of the standard model equation:

$$U_j\frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(C_\varepsilon\frac{k}{\varepsilon}\overline{u_j u_l}\frac{\partial \varepsilon}{\partial x_l}\right) + C_{\varepsilon_1}\frac{\varepsilon}{k}\mathcal{P} - C_{\varepsilon_2}\frac{\varepsilon^2}{k} \quad (5)$$

The coefficients of this equation depend on the model used and are here assigned the values recommended by their originators (see Table 1).

The modeled turbulence equations were transformed into cylindrical-polar coordinates and solved simultaneously with the mean continuity and momentum equations using a standard, finite-volume numerical method for boundary-layer flows. In this procedure, the solution is started from assumed or, if available, measured initial conditions and then advanced step by step in the predominant direction of the flow. A number of iterations were performed at each streamwise location to

Model	C_{ε_1}	C_{ε_2}	C_ε
LRR1	1.44	1.90	0.15
LRR2	1.45	1.90	0.18
SSG	1.44	1.83	0.183

Table 1: Coefficients of the ε - equation

facilitate coupling of the velocity and pressure fields. It should be mentioned that the radial velocity was obtained by integration of the continuity equation while the pressure gradients were obtained by integration of the radial momentum equation. The shear-stress components $\overline{uv}$ and $\overline{vw}$ were solved at grid locations displaced by half a cell from those of the mean velocities. This was done to prevent uncoupling of the velocity and turbulence fields. At the free stream all the dependent variables were set equal to zero; at the axis of symmetry the radial gradients of streamwise velocity and the normal stresses were set equal to zero while the transverse velocity and the shear stresses were themselves set equal to zero.

3 COMPARISONS WITH MEASUREMENTS

We apply the models first to the two-dimensional plane and axisymmetric jets (see Figure 1) discharged into stagnant surroundings. The calculations were started at the nozzle exit with assumed profiles and continued until the computed solutions ceased to change with downstream distance. The computational grid which expanded to match the growth of the shear layer consisted of 34 nodes in the cross-stream direction. The forward step was limited to 1 % of the local shear-layer half-width (defined as the point where the velocity falls to half of its centerline value). The predicted and measured plane jet spreading rates are compared in Table 2. There is considerable scatter in the measured and computed values alike due, for the most part, to the non-attainment of self-preservation or to insufficient grid resolution (Launder and Morse used a forward step of 5 % of the local half-width and attributed the difference with the Launder et al. (1975) result to the "larger forward steps" taken in the latter). The predicted and measured mean and turbulence profiles are compared in Figure 2. Overall, the SSG model appears to underestimate the measured turbulent stresses which is consistent with the low spreading rate obtained with this model. The reason for this result can clearly be seen from an inspection of the term $-C_1^*\mathcal{P}b_{ij}$ which has no counterpart in the LRR models. This term, while fairly uninfluential for the normal stresses, makes a large and negative contribution to the shear stress equation leading to the reduction of this quantity across the shear layer. A drop in the level of the shear stress reduces the rate of production of turbulent kinetic

	$\frac{dy_{1/2}}{dx}$
Measurements	**0.102[1] , 0.103[2,3] , 0.11[4]**
LRR1	**0.122 (*present*), 0.116[5] , 0.123[6]**
LRR2	**0.108 (*present*), 0.116[5]**
SSG	**0.092**

[1]Gutmark (1970); [2]Patel (1970); [3]Robins (1971); [4]Haskestad (1965); [5]Launder et al. (1975); [6]Launder and Morse (1979)

Table 2: Predicted and measured plane-jet spreading rates

energy and this in turn lowers the normal Reynolds stresses, either directly through the production term of $\overline{u^2}$ or indirectly through reduced transfer of energy via the pressure-strain correlation. It is nevertheless quite encouraging to see that the SSG model yields results that are comparable with the data considering that they were not used in its calibration. The model reproduces the observed turbulence anisotropy on the jet's axis and, in particular, correctly predicts $\overline{w^2}$ to be higher than $\overline{v^2}$ in contrast with LRR2 which predicts them to be equal.

Following Launder and Morse, we checked the sensitivity of the SSG model to C_s by repeating the calculations with this coefficient reduced by 10% of its original value. The effects of this change are to improve the prediction of the spreading rate by about 3% to 0.0946 but only at the expense of causing the turbulent stress profiles to decay much faster to their free-stream levels. Reducing C_ε by a similar amount increases the spreading rate by a mere 0.3 %. These results are very similar to the percentage responses obtained by Launder and Morse and are taken here to mean that the model's performance cannot be significantly improved by fine-tuning the coefficient of the Daly and Harlow model.

Attention is next turned to the axisymmetric jet whose predicted and measured spreading rates are compared in Table 3. All the models overestimate this quantity by quite a margin, though the mechanism which has led to the SSG model's underprediction of the plane-jet value is seen here to bring about a closer agreement with the data. The improved agreement is, of course, quite fortuitous but it lends support to the belief that the plane jet/round jet anomaly is not resolvable by further refinement of the pressure-strain model. The SSG model's results for the mean and turbulent profiles, shown in Figure 3, are broadly the same as the two LRR models.

It has already been mentioned that the plane and axisymmetric jets have not been used in calibrating the coefficients of the SSG model. That was done solely by reference to simple homoge-

	$\frac{d r_{1/2}}{d x}$
Measurements	0.086[1], 0.094[2]
LRR1	0.145 (*present*), 0.135[3], 0.148[4]
LRR2	0.121
SSG	0.112

[1]Rodi (1972); [2]Hussein et al. (1994); [3]Launder and Morse (1979)
[4]Musonge (1983)

Table 3: Predicted and measured axisymmetric-jet spreading rates

neous shear flow and to grid turbulence. This contrasts with the approach of Launder et al. (1975) and Gibson and Younis (1986) who took the fundamental flows to provide only a rough guide to the coefficients which were then refined by computer optimization involving a wide range of flows including the plane and axisymmetric free jets.

Attention is turned next to the swirling axisymmetric jet studied experimentally by Morse (1980). This experiment remains the best documented test case for this flow and has recently formed part of the benchmark flows chosen for turbulence-model assessments (Bradshaw et al. 1994). The experiments were performed for two values of the Swirl Number, while the comparisons made here are for the greater of the two (S=0.40) which demonstrated the best internal consistency and where the effects of swirl are most pronounced. The present calculations were started from x/D = 0.5, using the measured mean-velocity and turbulence profiles at this first transverse position. The starting profile for ε was deduced by inversion of the usual eddy-viscosity relation following the practice of Launder and Morse and of Gibson and Younis (1986). The computations were performed on a grid consisting of 34 cross-stream nodes with a forward step limited to 2.0 % of the local half width.

We reverse the usual order for presentation of the computed results by starting with the turbulent stresses and in particular the component $\overline{uw}$ obtained by Launder and Morse to be of the wrong sign. Figure 4 compares the predicted and measured profiles of this quantity. The profiles obtained with the LRR1 model correspond very closely to those reported by Launder and Morse using the same model. This is very gratifying considering that very different codes were used and bearing in mind the scatter observed in the prediction of non-swirling flows. The quadratic model suffers the same problem as the two linear ones in producing negative values in the region close to the axis ($r/x < 0.1$); however, it is then found to recover, producing positive stress values in the outer parts. In contrast, the linear models predict this quantity to be largely zero (LRR2) or negative (LRR1) throughout the flow. This is a very important result as it will determine the subsequent development of the flow. Its causes can be traced to the form of the pressure-strain models which for $\overline{uw}$ are reproduced below:

$$(\Phi_{\overline{uw}})_{LRR1} = \underbrace{-3.0\varepsilon b_{13} + 1.53 b_{23}\frac{\partial U}{\partial r} + 1.53 b_{12}\frac{\partial W}{\partial r} - 0.22 b_{12}\frac{W}{r}}$$

Negative in the outer layer ($W > W_m$)

$$(\Phi_{\overline{uw}})_{SSG} = \overbrace{-(3.4 + 1.8\frac{\mathcal{P}}{\varepsilon})\varepsilon b_{13} + 0.825 b_{23}\frac{\partial U}{\partial r} + 0.825 b_{12}\frac{\partial W}{\partial r} - 0.425 b_{12}\frac{W}{r}} \tag{6}$$

$$+$$

$$4.2\{(b_{11} + b_{33})b_{13} + b_{12}b_{23}\}\varepsilon$$

It should first be pointed out that at the starting station, x/D=0.5, $\overline{uw}$ was obtained to be negative in the inner layer and positive in the outer layer in between the maximum tangential velocity and free stream. Focusing attention on the outer layer where the differences between the various models are largest, it is immediately clear that all terms in the LRR1 model act as sinks for $\overline{uw}$ which consequently becomes negative (note that only a small part of this component's production rate is positive). In the SSG model, the terms that are counterparts to LRR1 are also always negative, but to a smaller extent as can be seen from the values of the multiplying coefficients. However, the quadratic terms are positive in the outer layer and it is those that are responsible for maintaining $\overline{uw}$ positive in the outer layer.

The consequences of this result on the development of the swirling jet cannot be understated. This can be seen in Figure 5 where the measurements and predictions of $\overline{uv}$ are compared. The presence of the term 2 $\overline{uw}$ W/r in the $\overline{uv}$ transport equation accounts for the differences between the model predictions. A higher value of $\overline{uv}$ is associated with a more rapid expansion of the shear layer and, from continuity, to a faster decay of velocity. Note that the differences between the LRR models and the SSG model are greatest at the early stages of the jet development where the effects of swirl are most pronounced. Downstream, where $\overline{uw}$ will have decayed to about 10 % of its initial value, the LRR models predict higher $\overline{uv}$ levels - a consequence of their overestimating this quantity in the non-swirling case (Figure 3).

In Figure 6 it can be seen that the shear-stress component $\overline{vw}$ which is responsible for the radial diffusion of tangential momentum is also better predicted by the SSG model, particularly at the early stages of development. The predicted and measured normal stresses are compared in Figures 7-9. The behavior of those quantities and the relative performance of the different models can be explained by the appearance in their transport equations of the shear-stress components already presented. Thus, for example, the higher levels obtained for $\overline{u^2}$ by the SSG model are due to the higher values obtained for $\overline{uv}$ which enters into its production.

The predicted and measured profiles of the axial and tangential mean velocities are compared in Figures 10 and 11. The shapes of the predicted profiles are determined by the distributions of $\overline{uv}$

and $\overline{vw}$ with the fuller profiles obtained using the SSG model corresponding to the higher values of the turbulent shear stresses obtained with that model. Finally, the predicted and measured decay of the maximum axial and tangential velocities in the streamwise direction are compared in Figure 12. Also plotted there are the computed and measured development of the jet's half width. This parameter is of primary practical interest and is perhaps the most sensitive measure of a model's suitability for swirling-flow calculations. Only the SSG model manages to capture the observed rapid growth of this quantity in the initial region where the effects of swirl are most pronounced. That this should be the case follows from the fact that higher values of $\overline{uv}$ were produced as a result of the quadratic model's better prediction of the sign and magnitude of $\overline{uw}$. The linear models eventually produce a jet half-width value comparable to that measured - but that only occurs far downstream from the nozzle exit, where the importance of swirl has diminished, and the models' tendency to overestimate the spreading rate of the *non-swirling* jet (see Table 3) comes into play.

4 CONCLUSIONS

The performance of the SSG model was tested in the benchmark plane and axisymmetric free jets without swirl, given that neither flow entered into the calibration of the model. Results that were on balance comparable to those of the linear LRR models were obtained. While the SSG model underpredicts the spreading rate of the plane jet by 12%, it overpredicts the axisymmetric value by only 19%: an improvement on the linear models where the overprediction is 30% or more. The results for the swirling jet obtained from the SSG model constitute a substantial improvement over those obtained with the linear LRR models. Most notably, the shear stress component $\overline{uw}$ was found to have generally the correct sign resulting in better overall predictions, especially for the growth rate of the swirling jet. This is in sharp contrast to the linear models which predict lower growth rates, with the differences between the two modeling approaches attributed directly to the contribution made by the quadratic slow term in the SSG pressure-strain model. It is interesting to note that the same quadratic term has recently been shown by So et al. (1994) to be essential for the correct prediction of the von Karman constant in flat-plate boundary layer flows when integration of the model is carried out through the viscous sublayer directly to the wall. It is likely that these terms will also improve the prediction of complex wall-bounded turbulent flows and, therefore, testing of the SSG model in a number of such flows will be the subject of a future study.

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LIST OF FIGURES

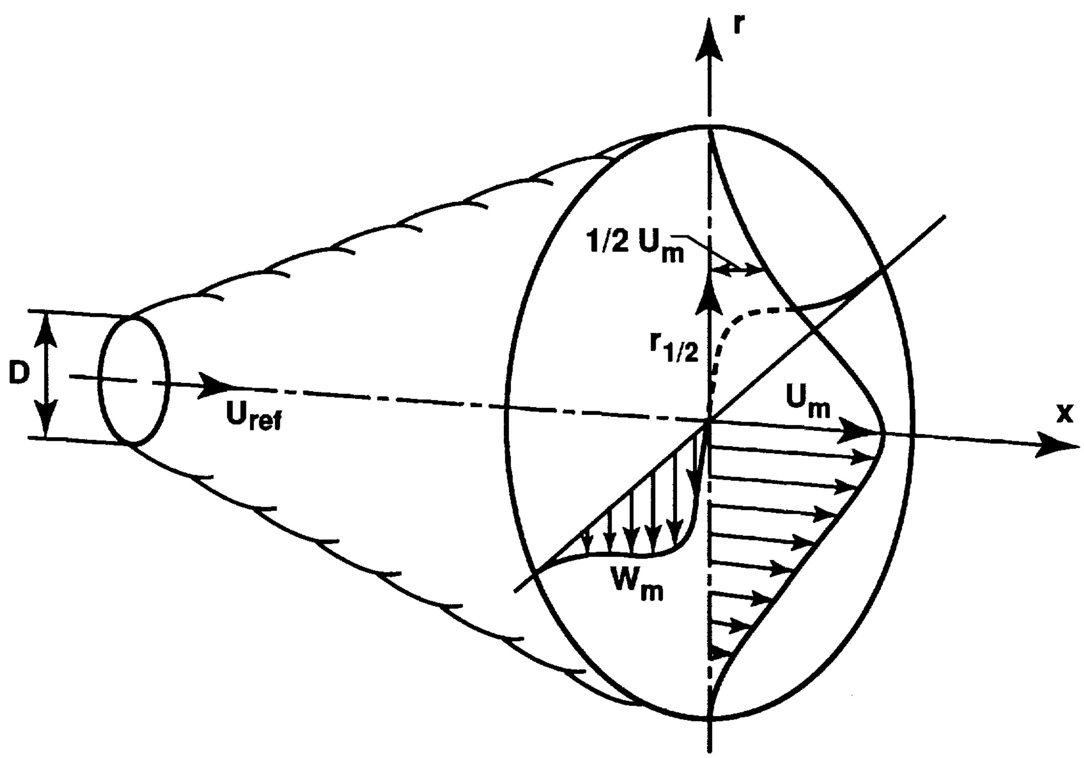
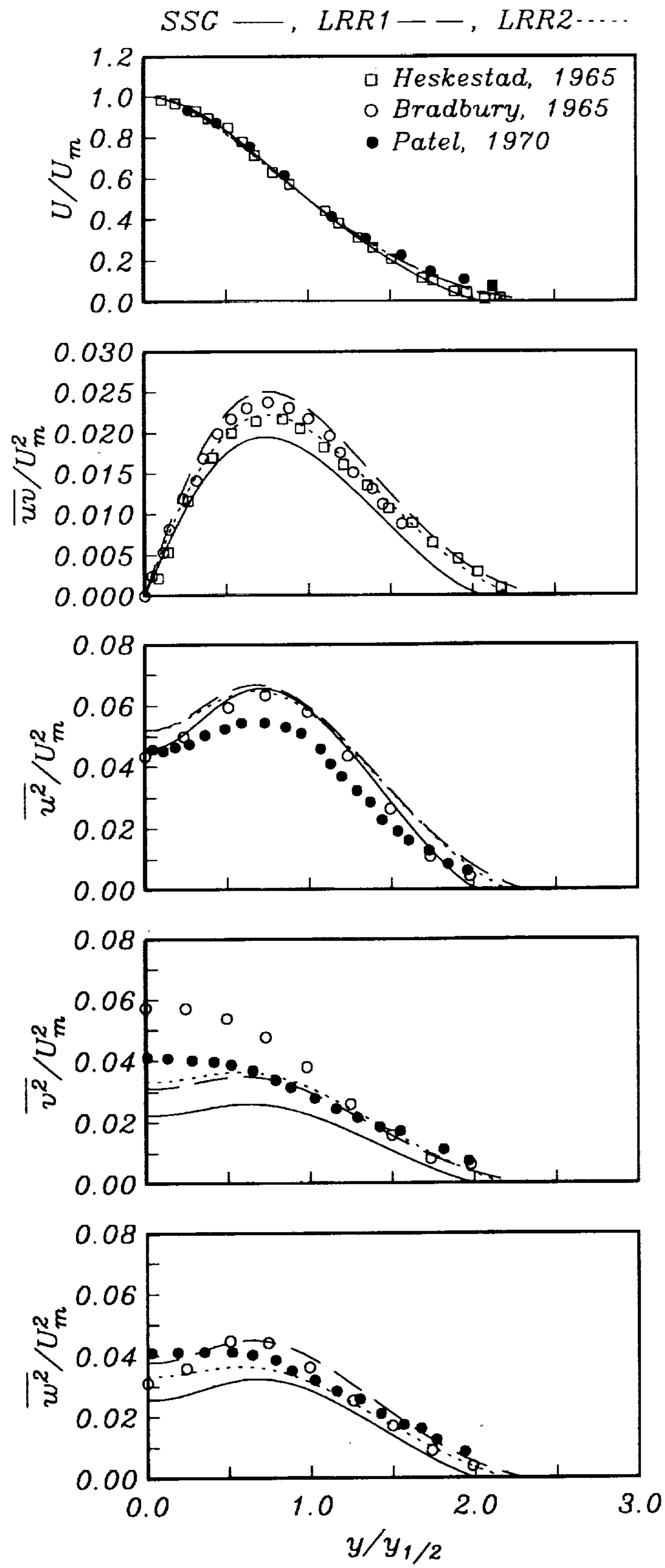


Figure 1: The swirling free jet. Coordinates and notation.



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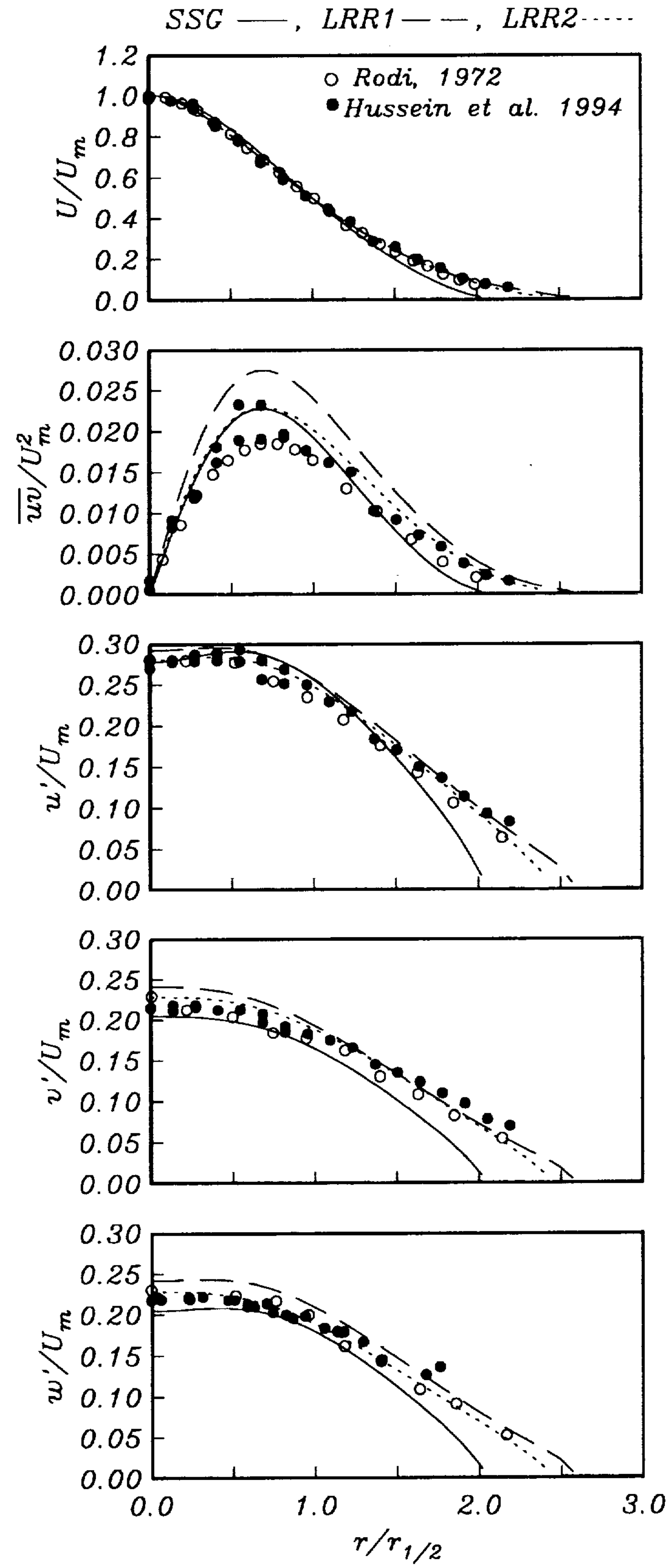

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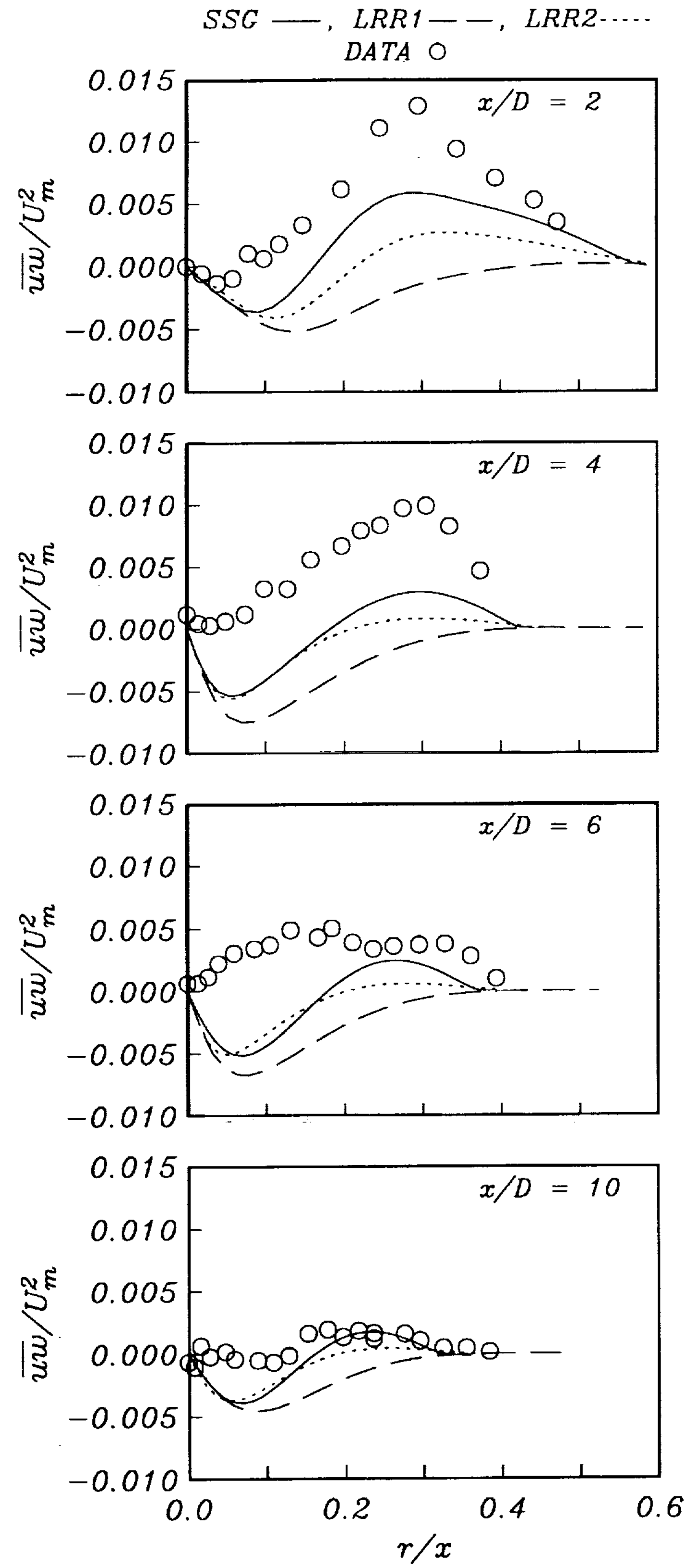

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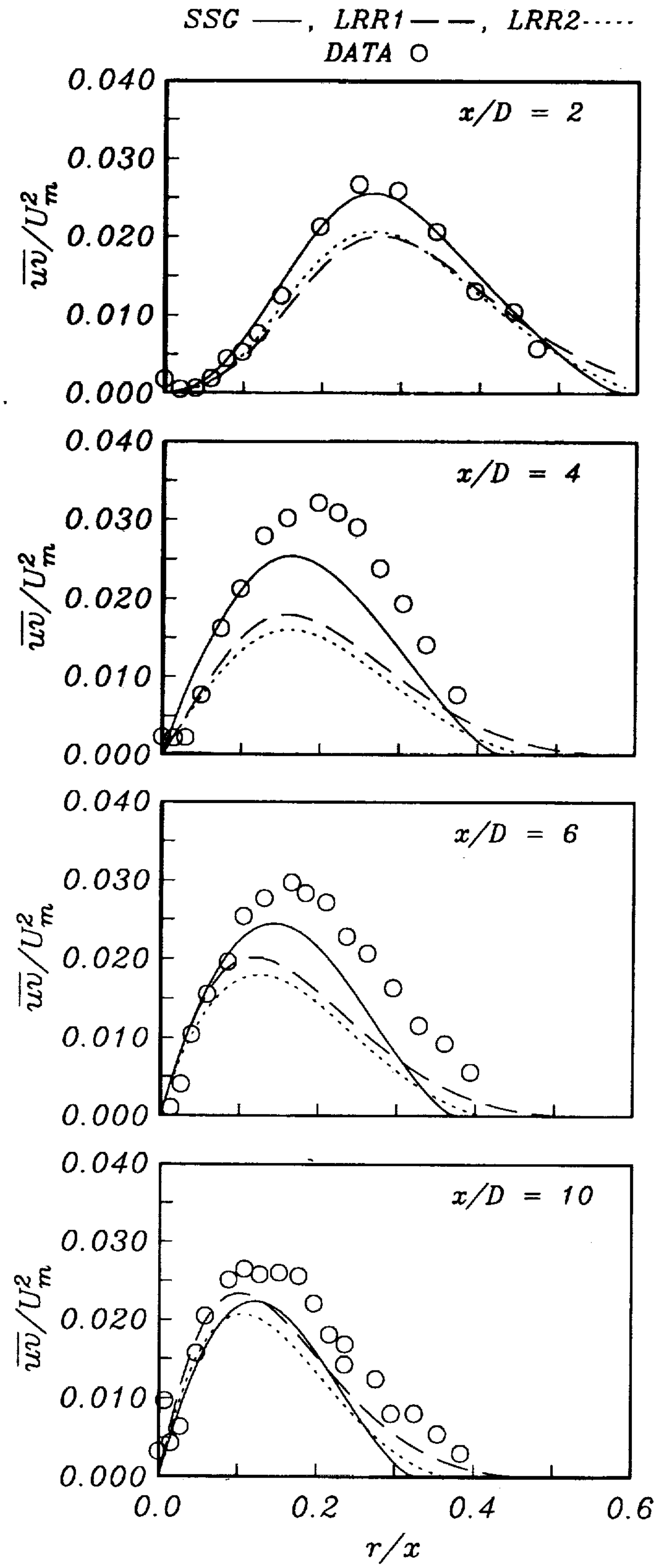

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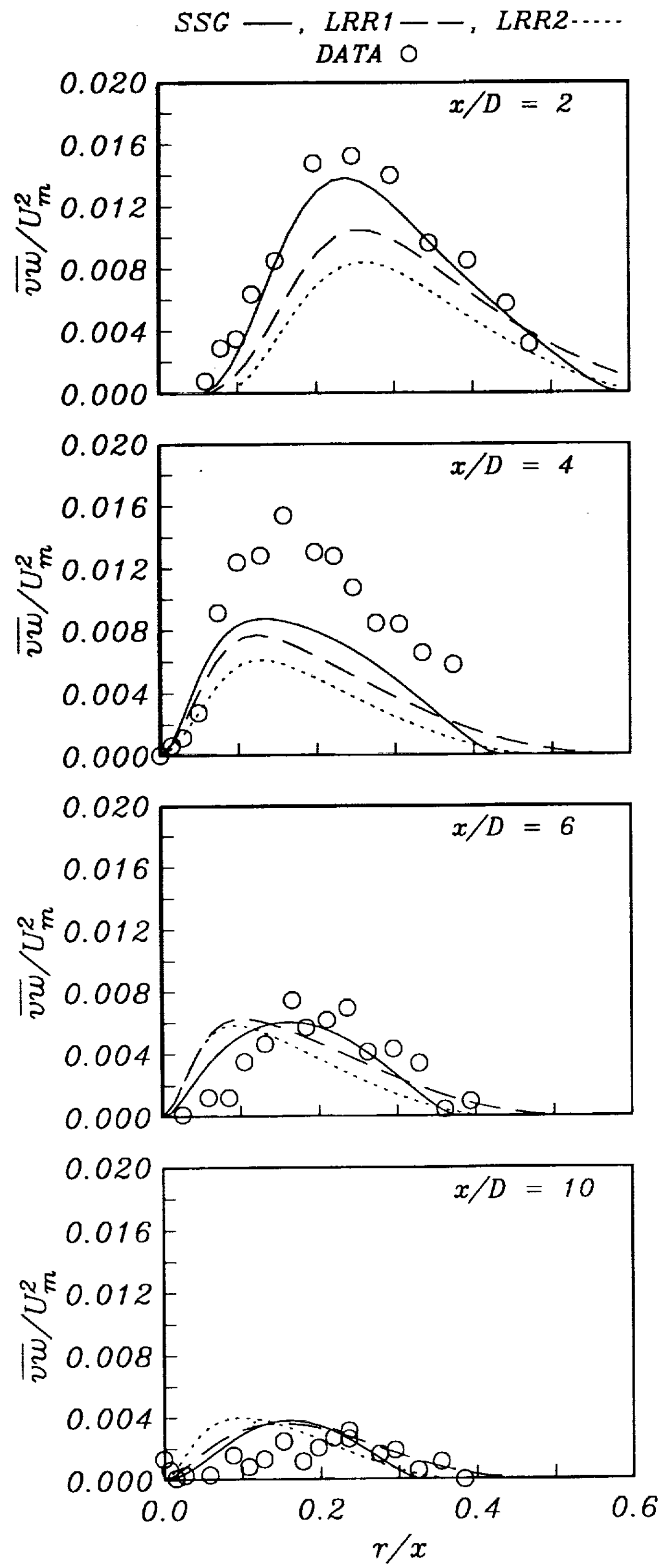

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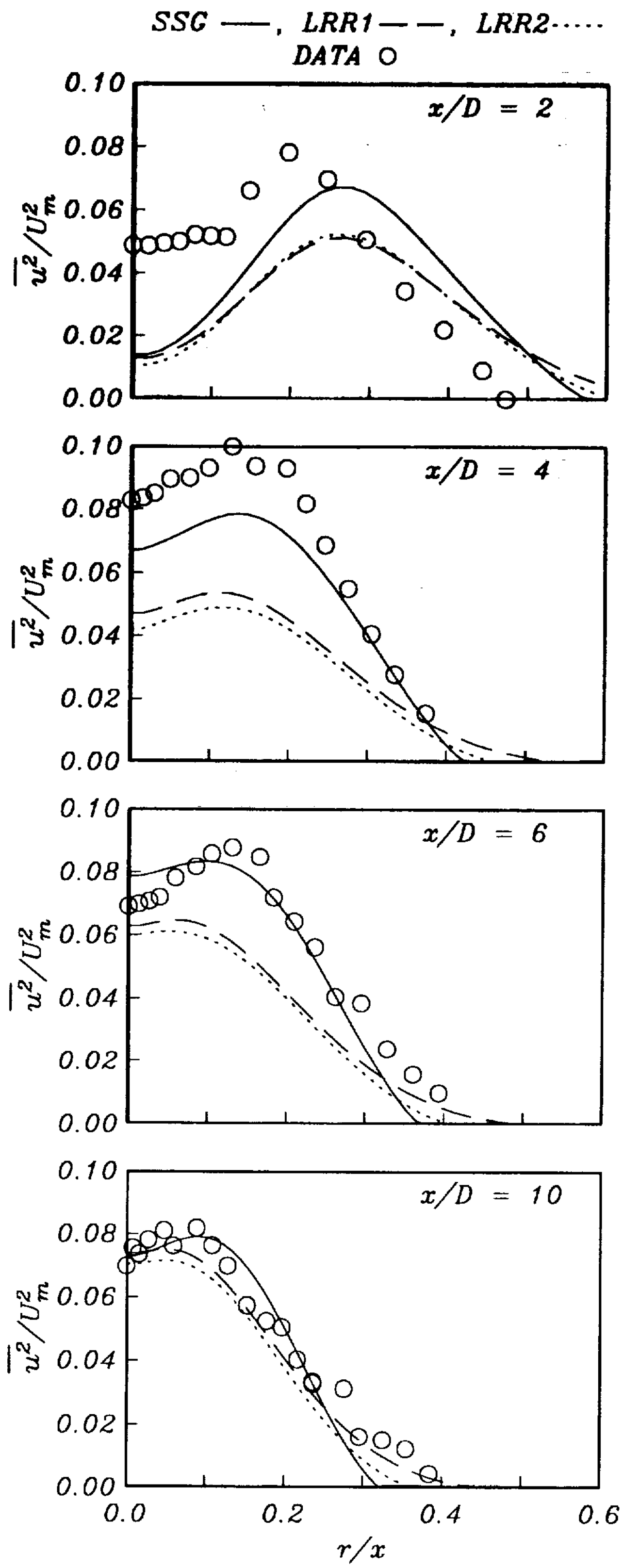

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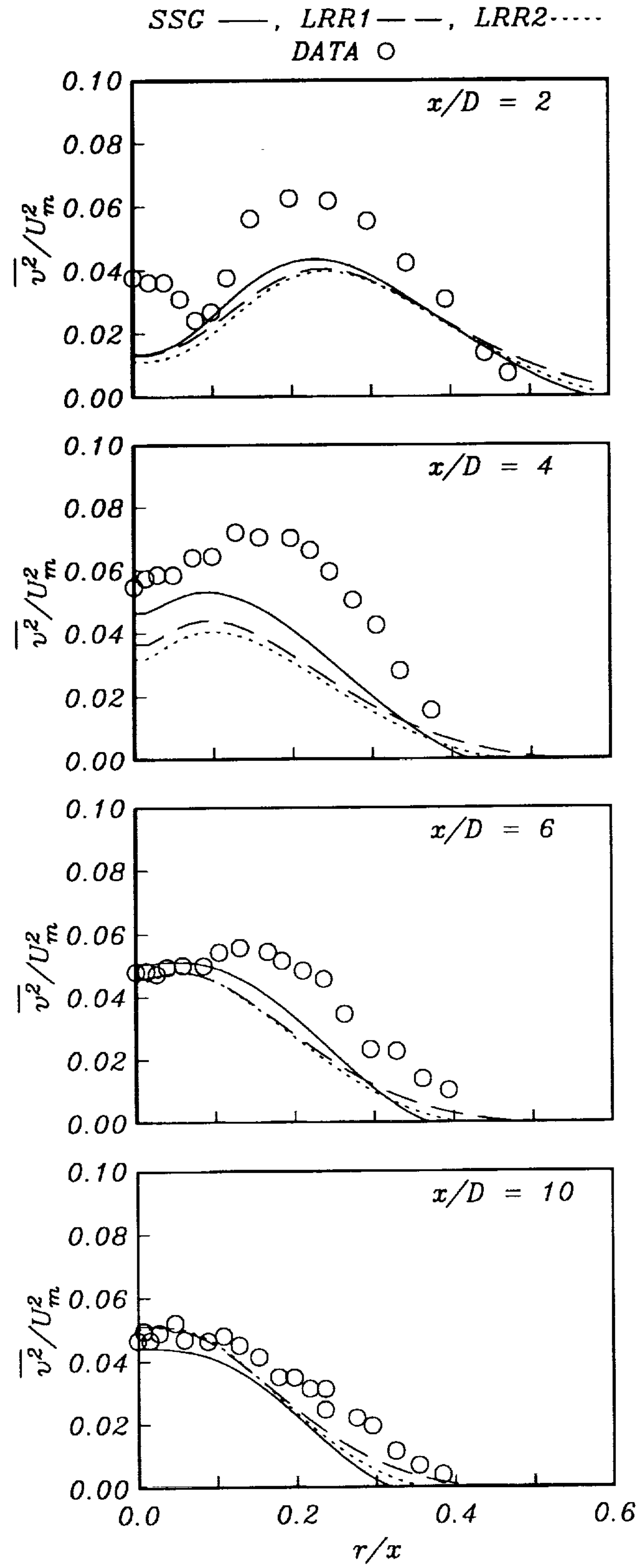

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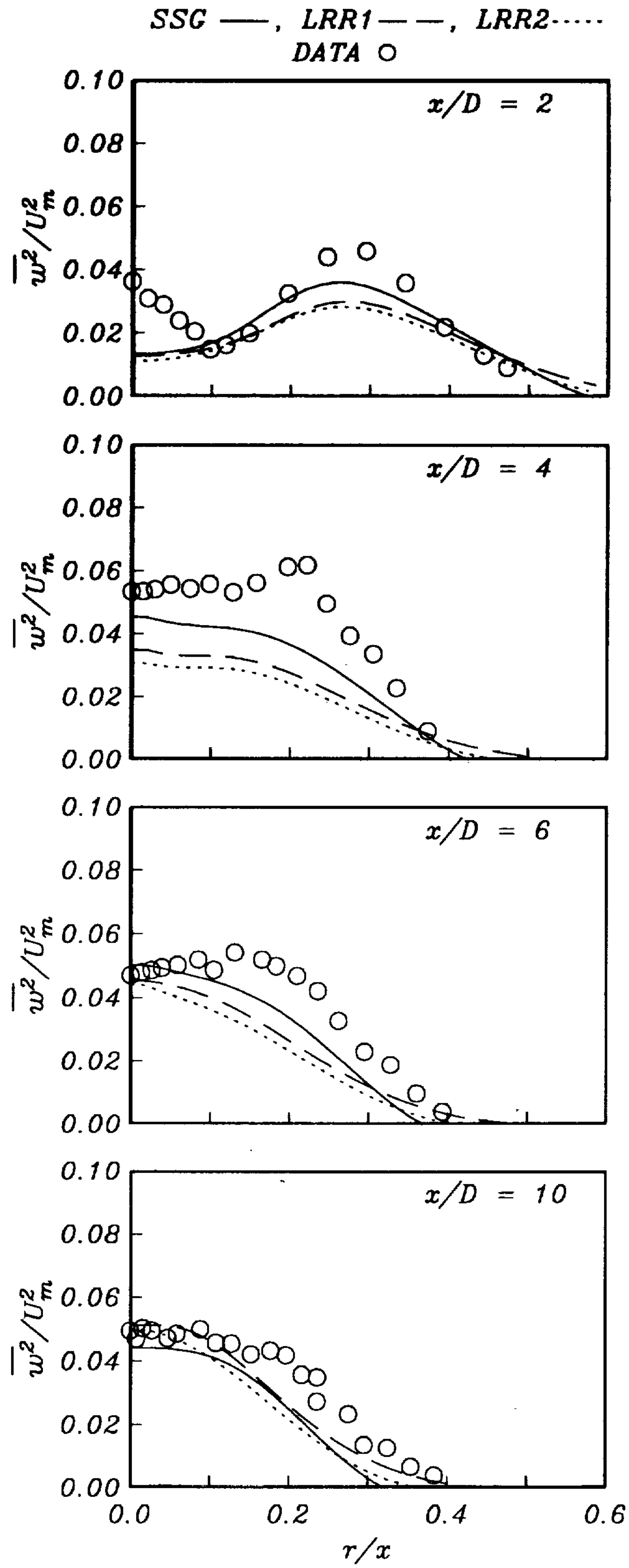

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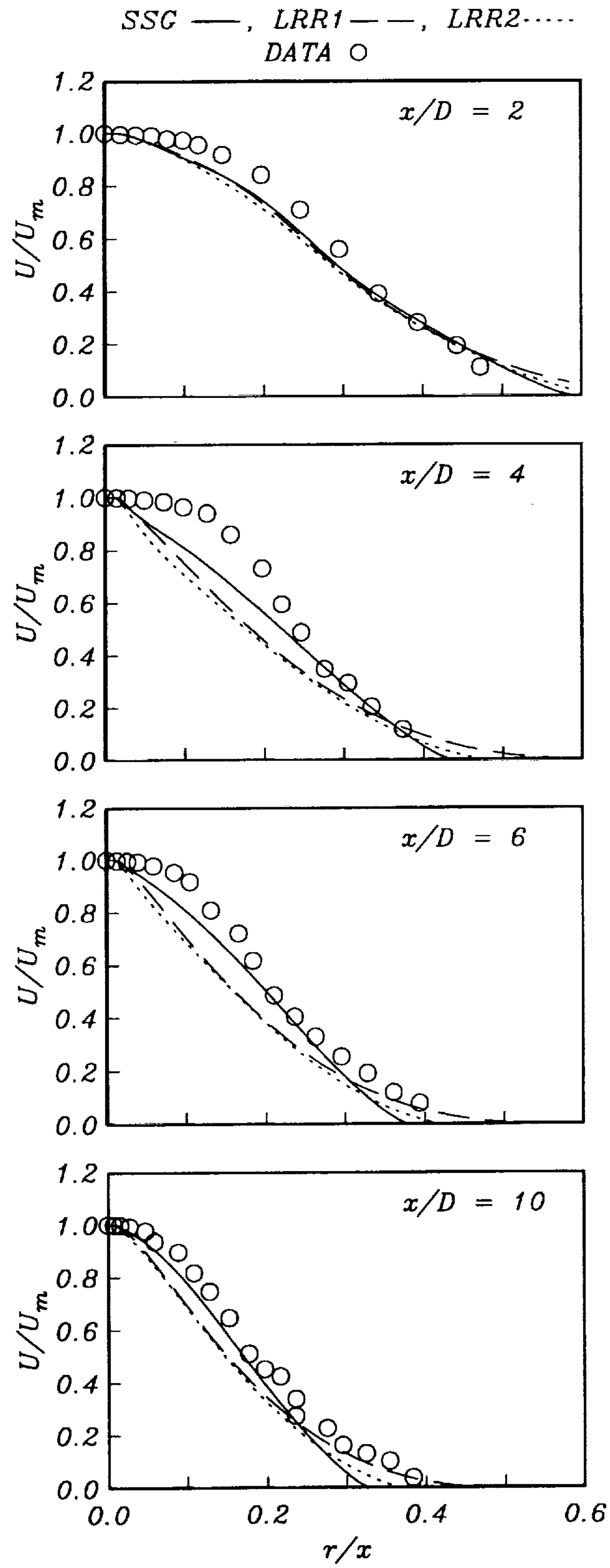

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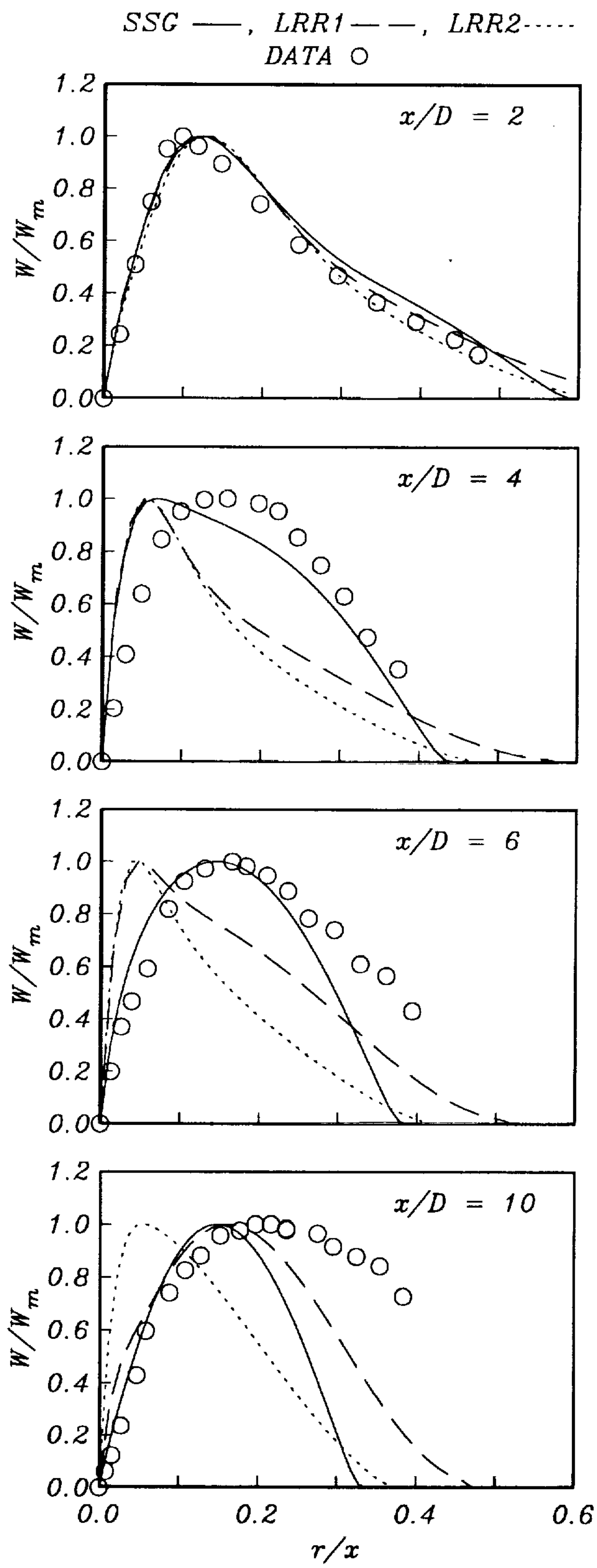

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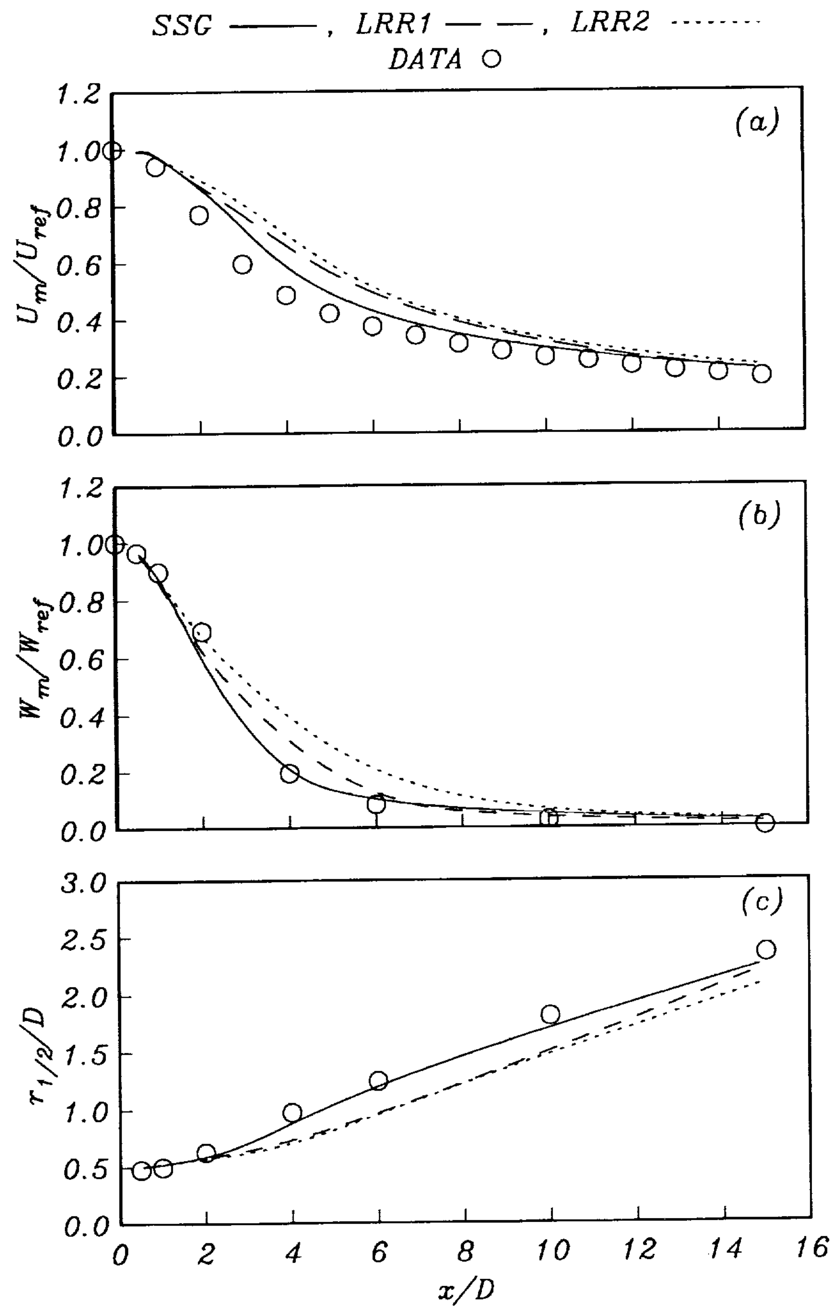

Figure 12: Predicted and measured streamwise variation of a): center-line axial velocity; b) maximum swirl velocity; c) jet's half-width. component $\overline{u^2}$.

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13. ABSTRACT *(Maximum 200 words)*
Data from free turbulent jets both with and without swirl are used to assess the performance of the pressure-strain model of Speziale, Sarkar and Gatski which is quadratic in the Reynolds stresses. Comparative predictions are also obtained with the two versions of the Launder, Reece and Rodi model which are linear in the same terms. All models are used as part of a complete second-order closure based on the solution of differential transport equations for each non-zero component of $\overline{u_i u_j}$ together with an equation for the scalar energy dissipation rate. For non-swirling jets, the quadratic model underestimates the measured spreading rate of the plane jet but yields a better prediction for the axisymmetric case without resolving the plane jet/round jet anomaly. For the swirling axisymmetric jet, the same model accurately reproduces the effects of swirl on both the mean flow and the turbulence structure in sharp contrast with the linear models which yield results that are in serious error. The reasons for these differences are discussed.

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